DOCUMENT RESUME

ED 048 371 TM 000 463

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TITLE Comparison of Rank Analysis of Covariance and Nonparametric Randomized Blocks Analysis.
PUB DATE Jan 71
NOTE 35p.; Paper presented at the Annual Meeting of the American Educational Research Association, New York, New York, February 1971

EDRS PRICE EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS *Analysis of Covariance, *Analysis of Variance, *Goodness of Fit, Hypothesis Testing, *Nonparametric Statistics, Predictor Variables, *Research Design, Sampling, Statistical Analysis

ABSTRACT

The relative power of three possible experimental designs under the condition that data is to be analyzed by nonparametric techniques; the comparison of the power of each nonparametric technique to its parametric analogue; and the comparison of relative powers using nonparametric and parametric techniques are discussed. The three nonparametric techniques concerned are the Kruskal-Wallis test on data where experimental units have been randomly assigned to levels of the independent variable, Friedman's rank analysis of variance (ANOVA) on data in a randomized blocks design, and a nonparametric analysis of covariance (ANCOVA). The parametric counterparts are respectively; one way analysis of variance (ANOVA), two-way ANOVA, and parametric ANCOVA. Since the nonparametric tests are based on large sample approximations, the goodness of fit for small samples is also of concern. Statements of asymptotic relative efficiency and imprecision are reviewed as suggestive of small sample powers of the designs and statistics investigated. Results of a Monte Carlo investigation are provided to further illustrate small sample power and to provide data on goodness of fit. (Author/CK)

ED048371

COMPARISON OF RANK ANALYSIS OF COVARIANCE AND NONPARAMETRIC RANDOMIZED BLOCKS ANALYSIS

Paper presented at the 1971 AERA convention

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January 1971

TM 000 463

Consider the case of an experimenter who is interested in testing the hypothesis that t levels of an independent variable have an identical effect on the dependent variable values of the units in a population. Given that he has unbiased estimates of treatment effects, the experimenter should choose for a fixed sample size a procedure that provides as statistically powerful a test as possible. The purposes of the present paper are to discuss the relative power of three possible designs under the condition that data are to be analyzed by nonparametric techniques, to compare the power of each nonparametric technique to the power of its parametric analogue, and to compare the relative powers using nonparametric techniques to the better known relative powers using parametric techniques. The three nonparametric techniques are the Kruskal-Wallis test on data where experimental units have been simple random assigned to levels of the independent variable, Friedman's rank ANOVA on data in a randomized blocks design, and a nonparametric analysis of covariance (ANCOVA) which has recently been proposed by Quade (1967). The parametric analogues are respectively; one way analysis of variance (ANOVA), two way ANOVA, and parametric ANCOVA. Since the nonparametric tests are based on large sample approximations, the goodness of their fit for small samples is also of concern. Statements of asymptotic relative efficiency and imprecision are reviewed as suggestive of the relative small sample powers of the designs and statistics investigated. Results of a Monte Carlo investigation are provided to speak more directly to the concern of small sample power and to provide data on goodness of fit.

In a 1970 AERA paper (Porter and McSweeney, 1970) we empirically investigated the small sample goodness of fit and power of the Kruskal-Wallis and Friedman tests, but did not provide empirical α's and powers for their parametric analogues.

Our 1970 AERA paper also investigated the effects of analyzing data in randomized block form with a Kruskal-Wallis test, and the effects of using Friedman's test on data where blocks were formed after initial random assignment. Here we extend last year's work by considering Quade's nonparametric ANCOVA, comparing the small sample properties of nonparametric ANCOVA to our last year's data on Kruskal-Wallis and Friedman's tests, and by presenting empirical results for all three parametric analogues. Partial motivation for our study is provided by Quade (1967, p. 1198) when in reference to his nonparametric ANCOVA he states "Another disadvantage of the rank method at this time appears to be its unknown behavior for small samples, say with less than five to ten observations per group." Further motivation is provided by the apparent lack of information in the literature about the small sample power of different experimental designs for testing a common hypothesis when nonparametric tests are used. By comparison considerable attention has been given to study of the relative small sample power among those same experimental designs when parametric tests are used. In another 1971 AERA paper (McSweeney and Porter, 1971) we further extend our considerations of small sample goodness of fit and power to include a modification of Quade's nonparametric rank ANCOVA and a nonparametric index of response which we propose.

When an experimenter is interested in testing the above stated hypothesis that t levels of an independent variable have an identical effect on the dependent variable values of the units in a population, he can proceed by obtaining a simple random sample from the population of interest and then use simple random assignment of experimental units to levels of the independent variable. Given the assumptions of normality, homoscedasticity and independence he can analyze his data by a one-way ANOVA. However, if antecedent information is available on a concomitant variable and if the concomitant variable has a linear relationship with the dependent variable, ANCOVA will probably provide a more powerful test. The well known assumptions for parametric ANCOVA are conditional normality, conditional homoscedasticity, independence, and parallel treatment group regression lines.

Another method of using antecedent information on a concomitant variable in an attempt to improve statistical power is to employ a randomized blocks design. Pingel (1969) defines two types of randomized blocks design and compares their precisions when data are analyzed by a two way ANOVA. Unfortunately, the more precise method involves forming blocks on the population distribution of the concomitant variable and thus requires more information than is typically available. The other type of randomized blocks design requires a random sample of tb units from the population, which are then put in rank order on the basis of their concomitant variable values. Then the experimenter randomly assigns one unit to each level of the independent variable from the first set of t units, one unit to each level from the second set of t units, and so on until the tb observations have all been assigned. The second type of randomized blocks design is the one investigated in the present study because of its greater feasibility for an experimenter. The assumptions for two way ANOVA on data in a randomized blocks design are normality, homoscedasticity, independence, and no block by treatment interaction.

Over the past several years there has been some debate as to the general utility of nonparametric techniques. The robustness of the analysis of variance F test (Box, 1955) has been offered as an argument in favor of its use, even when the assumptions of normality and homoscedasticity are violated. This argument emphasizes the goodness-of-fit of the F test statistic to the theoretical null distribution even when the assumptions for the F test are seriously in error. Its principal concern is with the size of the actual Type I error. Despite the good control over Type I error given by the F test when parametric assumptions are violated, nonparametric tests for the same designs are generally more powerful than their parametric counterparts in these circumstances (Bradley, 1968). Therefore, the Kruskal-Wallis, Friedman's ANOVA, and Quade's nonparametric ANCOVA are to be preferred over their parametric counterparts when parametric assumptions have been violated.

Both the Kruskal-Wallis and the Friedman test statistics operate on ranks, but they differ in their methods of ranking. When calculating a Kruskal-Wallis test statistic the units are ranked 1 through tb on the basis of their dependent variable values. The Friedman test statistic requires that the units within each block be ranked 1 through t on the basis of their dependent variable values. The Kruskal-Wallis test statistic is then defined

$$H = \frac{12}{tb^2(tb+1)} \sum_{T=1}^{t} R_T^2 - 3(tb+1),$$

and the Friedman test statistic is defined

$$\chi_r^2 = \frac{12}{tb(t+1)} \sum_{T=1}^{t} R_T^2 - 3b(t+1),$$

where R_T denotes the sum of the b ranks under the Tth level of the independent variable. The assumptions for the Kruskal-Wallis test are independence of units and measurement of the dependent variable on at least ordinal scale. The assumptions for the Friedman test are independence among blocks and that within each block the dependent variable is measured on at least an ordinal scale.

Because the nonparametric ANCOVA proposed by Quade is relatively new and therefore probably not well known, a more detailed description is appropriate. Although Quade's nonparametric ANCOVA is not restricted to use of a single concomitant variable, the present paper considers only the single concomitant variable case. The restriction is made to facilitate comparison with our earlier investigation of Friedman's ANOVA on data in a randomized blocks design where we considered only a single concomitant variable. The nonparametric analysis of covariance test statistic is calculated by first replacing each observation on the concomitant or covariable by its respective rank and each observation on the dependent variable by its respective rank, where in both cases ranking is done on all experimental units across all levels of the independent variable. Each rank on the covariable is then replaced by its deviation from the mean of ranks on the covariable, and each rank on the dependent variable is replaced by its deviation from the mean of the ranks on the dependent variable. Disregarding levels of the independent variable, least squares theory is used on the deviation data to provide predicted deviations on the dependent variable. Because both sets of deviation data have a mean of zero and common variance, the least squares regression equation is

$$\hat{d}_{Y_{TB}} = r_S d_{X_{TB}} ,$$

where r_S is the Spearman rank order correlation coefficient calculated on the original rank data ignoring levels of the independent variable,

d denotes a deviation rank,

X denotes the covariable,

Y denotes the dependent variable,

T denotes the Tth level of the independent variable,

and B denotes the Bth replication within the Tth level of the independent variable.

A new variable, say Z_{TB}, is formed by subtracting each unit's predicted deviation from its observed deviation,

$$Z_{TB} = d_{Y_{TB}} - \hat{d}_{Y_{TB}} .$$

The new variable, Z_{TB}, becomes the dependent variable for a one-way ANOVA F test. The reference distribution for the F test statistic is the central F with degrees of freedom equal to t - 1 and tb - t as tb $\rightarrow \infty$.

The only assumption that Quade's nonparametric ANCOVA has in common with parametric ANCOVA is independence of units. The nonparametric ANCOVA further assumes that the covariable has the same distribution in each population. The identical covariable distributions assumption is necessarily met if observations on the covariable are taken prior to the experiment and if simple random assignment of units to levels of the independent variable is employed. The relaxation of the need for a linear relationship between the dependent variable and covariable to only a need for a monotonic relationship for nonparametric ANCOVA may be of particular practical importance. It is also helpful to note that Quade's nonparametric ANCOVA reduces to a Kruskal-Wallis test when the Spearman rank order correlation is zero.

The question of goodness of fit of the three nonparametric test statistic's sampling distributions to their large sample approximations must be answered empirically. Their relative powers and comparisons of their powers to the powers of their respective parametric analogues for small samples must also be answered largely on an empirical basis; however, statements of Pitman asymptotic relative efficiency are informative. Quade (1967) has shown that the asymptotic relative efficiency of his nonparametric ANCOVA with respect to the Kruskal-Wallis test is $1/(1-\rho_S^2)$ where ρ_S denotes the population Spearman correlation. Asymptotic relative efficiency statements for Friedman with respect to Kruskal-Wallis and nonparametric ANCOVA with respect to Friedman are not known.

When comparing the efficiency of nonparametric tests to their parametric counterparts given the parametric assumptions, Andrews (1954) has shown that the Kruskal-Wallis test has asymptotic relative efficiency of .955 with respect to the one-way ANOVA. Friedman (1937) has given the asymptotic relative efficiency of his test relative to the two-way ANOVA as .637 when t equals 2 and .912 when b equals 2. For other values of b and t the Friedman test has asymptotic relative efficiency between .637 and .912 (Noether, 1967). When the number of blocks for the Friedman test becomes large, its asymptotic relative efficiency with respect to the two-way ANOVA is .955 $\left(\frac{t}{t+1}\right)$. Quade (1967) demonstrated that his nonparametric ANCOVA has .955 $(1 - \rho_{XY}^2)/(1 - \rho_S^2)$ asymptotic relative efficiency with respect to the parametric ANCOVA, where ρ_{XY} denotes the population correlation of the dependent variable and the covariable and ρ_S is the population Spearman correlation. The relationship between the two coefficients is

$$\rho_{XY} = 2 \sin (\pi\rho_S/6),$$

given that X and Y have a bivariate normal distribution. It follows that the asymptotic relative efficiency of nonparametric to parametric ANCOVA is greatest when $\rho_{XY} = 0$ and becomes less as the absolute value of ρ_{XY} increases, having a smallest value of .866.

Cox (1957) provides indices of true imprecision for two-way ANOVA of data in randomized block form and parametric ANCOVA, which are instructive in choosing among the two techniques on the basis of their statistical power. Cox defines an index of true imprecision as the ratio of the average variance of the difference of means for pairs of levels of the independent variables over the average variance of the difference of means for pairs of levels of the independent variable if all of the variance of the dependent variable accounted for by the concomitant variable were removed. The index of true imprecision of two-way ANOVA on data in the type of randomized block form of interest is

$$1 + \frac{\rho_{XY}^2}{1 - \rho_{XY}^2} W,$$

where W is the expected mean square of X within blocks divided by the variance of X. The index of true imprecision for parametric ANCOVA is

$$\frac{tb - t - 1}{tb - t - 2} .$$

Based upon the above two statements of imprecision Cox concludes that where ρ_{XY} is less than .6, the randomized blocks design is to be preferred over ANCOVA; but where ρ_{XY} is equal to or greater than .8, ANCOVA is to be preferred over randomized blocks. These statements do not reflect the differences in number of degrees of freedom for the two parametric tests. When taken into consideration the differences in degrees of freedom suggest that a one-way ANOVA will be more powerful than either ANCOVA or randomized blocks, and that ANCOVA will be more powerful than randomized blocks when $\rho_{XY} = .0$.

Although the above statements of asymptotic relative efficiency and true imprecision provide no direct information about the small sample relative powers of the technique being investigated, they are suggestive. In general they suggest that given parametric assumptions the nonparametric techniques should be expected to yield slightly less power than their parametric analogues. Further, they suggest that as the absolute value of ρ_{XY} increases the power of the nonparametric ANCOVA should become progressively greater than the power of the Kruskal-Wallis. By a more indirect argument they suggest that Friedman ANOVA on data in randomized block form should be more powerful than the Kruskal-Wallis test for fairly large absolute values of ρ_{XY}. They are somewhat less suggestive of what to expect when comparing the small sample powers of Friedman's test to those for nonparametric ANCOVA.

Monte Carlo studies were conducted to provide information about the small sample properties of the sampling distributions of the Kruskal-Wallis and one-way ANOVA test statistics when calculated on data in a simple random assignment design; the nonparametric ANCOVA and parametric ANCOVA test statistics when calculated on data in a simple random assignment design with information on an antecedent concomitant variable available; and Friedman's ANOVA and two-way ANOVA when calculated on data in a randomized blocks design. Empirical estimates of the probability of a Type I error (α) provide information about the small sample goodness of fit for each of the three nonparametric statistics. Empirical estimates of power provide data relevant to choosing among the three nonparametric statistics, the three parametric statistics, and are indicative of the difference in power of nonparametric and parametric statistics when parametric assumptions are met.

Figure 1 presents an overview of the conditions under which each type of sampling distribution was investigated. For each X in Figure 1, sampling distributions for 1000 samples were generated for the central case, i.e. Ho: true, and one noncentral case, i.e. Ha: true. The population distributions indicated in Figure 1 were chosen to allow comparisons with the results from our earlier study of the Kruskal-Wallis and Friedman tests. Three values of t were investigated because of possible trends in the sampling distributions as the number of levels of the treatment independent variable increase. The smallest value of t was three since the Wilcoxon matched pairs test offers a more powerful alternative to the Friedman when t equals two. Three values of b were investigated because of possible trends in the sampling distributions as the number of units under each level of the independent variable increase. The smallest value of b was five since exact tests would be more appropriate for smaller values. Four values of the correlation between the concomitant variable and the dependent variable were investigated to provide information on the relative small sample powers of three nonparametric statistics, and to see if the interrelationships of their powers parallel the better known interrelationships of the powers of their parametric analogues.

Figure 1

CONDITIONS OF THE DESIGNS INVESTIGATED

		ρ_{xy}			
t	b	.0	.4	.6	.8
3	5			X	
	8	X	X	X	X
	10			X	
5	5				
	8			X	
	10				
8	5				
	8			X	
	10				

t = number of treatments

b = number of blocks

ρ_{xy} = the correlation between the concomitant variable and the dependent variable

X denotes a population investigated

The smallest value of the correlation was zero, defining the situation where the three designs are essentially the same; and the largest value was .8 since in practice correlations are infrequently larger. The non-central case was created by adding the value one to the dependent variable value of each unit under the first level of the treatment independent variable. For the three nonparametric tests the noncentral case was created prior to forming ranks. The non-central case was chosen because it seemed to represent a deviation from the null hypothesis that most experimenters would wish to notice. Further, it produced intermediate values of power which facilitated comparisons of the various techniques.

All data generation and subsequent calculations were done on a Control Data 3600 computer. A pseudo-random unit normal deviate was generated by first generating random numbers from a uniform distribution by the multiplicative congruent method. By the Central Limit Theorem the sum of sixteen of the above numbers is approximately normally distributed, and by a linear rescaling the sum becomes a pseudo-random unit normal deviate. In this way 1,000 random samples of size tb each were generated for each population indicated in Figure 1. Each population had a bivariate normal distribution with known correlation between the concomitant and dependent variables, and each marginal distribution normal zero, one. Given a sample of size tb, the data were then arranged into simple random assignment and randomized blocks designs.

Table 1 compares the empirical sampling distributions of the three nonparametric tests when calculated on data in their respective appropriate designs for $t = 3$, $b = 8$, and varying values of ρ_{XY}. The data for Kruskal-Wallis and Friedman tests are taken from our 1970 AERA paper (Porter and McSweeney, 1970). The three rows labeled central indicate the goodness of fit of the empirical distributions to their respective Chi Square and F reference distributions under the null hypothesis. The standard errors of the proportions representing empirical α's are approximately .009 for α = .10, .007 for α = .05, and .003 for α = .01. The empirical and nominal values of α are in close agreement for all three statistics with the greatest discrepancies at α = .10 for the Friedman. As noted in our 1970 paper the largest discrepancies for the Friedman are to be expected since the empirical values are closer to the values from the exact distribution of the Friedman test derived from permutation theory than are the nominal values. The discrepancy between the exact and large sample approximation distributions is reflecting the fact that the Friedman test statistic is a discrete variable with fewer possible values than the Kruskal-Wallis or nonparametric ANCOVA tests because of the difference in their methods of ranking.

The three rows labeled non-central contain the empirical values of power for the three nonparametric tests given that a common alternative hypothesis was true. Their largest standard error of approximately .016 occurs when power equals .5. As noted in our 1970 paper, the data clearly indicate that the Kruskal-Wallis is more powerful than the Friedman ANOVA when ρ_{XY} = .0. The data further indicate that nonparametric ANCOVA is more powerful than the Friedman ANOVA at all three levels of α when ρ_{XY} = .0. Consistent with asymptotic theory the data are not as clear when comparing nonparametric ANCOVA to the Kruskal-Wallis. Kruskal-Wallis is seen to be slightly more powerful at α = .10, .596 compared to .543; and α = .05, .437 compared to .418; but slightly less powerful than nonparametric ANCOVA for α = .01, .158 compared to .202. As ρ_{XY} increased the power of the Kruskal-Wallis remained relatively stable whereas the powers of Friedman ANOVA and nonparametric ANCOVA were both strict monotonic increasing functions of ρ_{XY} for all three levels of α. When $\rho_{XY} \geq .4$,

TABLE 1

EMPIRICAL PROPORTIONS OF KRUSKAL-WALLIS (KW), FRIEDMAN (Fr) AND NONPARAMETRIC ANCOVA (NC) TEST STATISTICS FALLING IN THEIR RESPECTIVE REGIONS OF REJECTION FOR α = .10, .05, .01; t = 3; b = 8; ρ_{xy} = .0, .4, .6, .8

	ρ_{xy} =	.0			.4			.6			.8		
	α	KW	Fr	NC	KW	Fr	NC	KW	Fr	NC	KW	Fr	NC
Central	.10	.102	.129	.097	.088	.116	.107	.093	.109	.103	.094	.129	.099
	.05	.053	.051	.053	.052	.047	.054	.039	.039	.058	.041	.057	.042
	.01	.005	.011	.012	.005	.005	.015	.004	.006	.015	.003	.009	.010
Non-central*	.10	.596	.528	.543	.550	.581	.594	.594	.687	.715	.562	.850	.914
	.05	.437	.332	.418	.391	.376	.467	.428	.457	.583	.408	.711	.831
	.01	.158	.125	.202	.145	.147	.229	.158	.212	.324	.149	.422	.590

*The non-central case was $\mu_1=1$, $\mu_2=\mu_3=0$

the power of nonparametric ANCOVA was greater than the power of Kruskal-Wallis and Friedman for all three levels of α. As noted in our 1970 paper, the data make no clear distinction on the basis of power between Kruskal-Wallis and Friedman ANOVA when correlation is .4. When the correlation is .6 or .8, Friedman was clearly more powerful than Kruskal-Wallis. There was a slight tendency for the nonparametric ANCOVA to become progressively more powerful than the Friedman ANOVA as ρ_{XY} increased. When the empirical power of the Friedman ANOVA is subtracted from that of the nonparametric ANCOVA for increasing values of ρ_{XY}, the differences for α = .10 are .015, .013, .028, .064; for α = .05 are .086, .091, .162, .120; and for α = .01 are .007, .082, .112, .168. The values of ρ_{XY} were .0, .4, .6, and .8 respectively.

Table 2 compares the empirical sampling distributions of the three parametric tests when calculated on data in their respective appropriate designs for $t = 3$, $b = 8$ and varying values of ρ_{XY}. As should be expected, the three rows labeled central indicate very good fit for all levels of α. The one exception is one-way ANOVA with ρ_{XY} = .6, which resulted in empirical α's .010 too large for all three nominal values of α.

The empirical powers indicate that when ρ_{XY} = .0, one-way ANOVA is more powerful than ANCOVA for all three levels of α. One way ANOVA is also more powerful than two-way ANOVA on data in randomized block form for α = .05, .458 compared to .453; and .01, .210 compared to .199; but less powerful for α = .10, .584 compared to .598. As ρ_{XY} increased the power of one-way ANOVA remained relatively stable, whereas the powers of two way ANOVA and ANCOVA were both strict monotonic increasing function of ρ_{XY} for all three levels of α. Perhaps the most interesting power comparisons are between two way ANOVA and ANCOVA. The empirical powers are only partially in support of Cox's (1957) statements cited earlier in the paper. For ρ_{XY} = .0 the comparisons are not of interest

TABLE 2

EMPIRICAL PROPORTIONS OF ONE AND TWO WAY ANOVA AND ANCOVA (PC) TEST STATISTICS FALLING IN THEIR RESPECTIVE REGIONS OF REJECTION FOR α = .10, .05, .01; t = 3; b = 8; ρ_{xy} = .0, .4, .6, .8

	ρ_{xy} =	.0			.4			.6			.8		
	α	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC
Central	.10	.102	.092	.104	.106	.096	.110	.124	.091	.101	.103	.097	.096
	.05	.052	.040	.054	.056	.055	.056	.060	.050	.052	.055	.049	.048
	.01	.011	.012	.011	.012	.012	.013	.020	.006	.014	.008	.010	.008
Non-central*	.10	.584	.598	.568	.575	.645	.625	.611	.736	.760	.646	.928	.957
	.05	.458	.453	.441	.439	.502	.504	.482	.606	.645	.509	.827	.894
	.01	.210	.199	.198	.215	.223	.239	.216	.320	.365	.238	.582	.688

*The non-central case was $\mu_1=1$, $\mu_2=\mu_3=0$

since one-way ANOVA is preferred. When $\rho_{XY} = .4$, two-way ANOVA was more powerful than ANCOVA when $\alpha = .10$, .645 compared to .625; but less powerful when $\alpha = .05$, .502 compared to .504; and $\alpha = .01$, .223 compared to .239. As ρ_{XY} increased from .4, ANCOVA increased in power at a faster rate than two way ANOVA for all three levels of α with the differential rate of increase in power being most noticable for $\alpha = .01$.

Taken together, Tables 1 and 2 afford a comparison of nonparametric techniques to their parametric analogues. As was expected, the empirical α's in Table 2 are in general slightly closer to their respective nominal values than are the empirical α's in Table 1. This finding reflects the discreteness of the nonparametric test statistics. As was suggested by the statements of asymptotic relative efficiency presented earlier, there was a greater loss in power going from the two way ANOVA to the Friedman than there was going from the one way ANOVA to the Kruskal-Wallis or from the parametric ANCOVA to the nonparametric ANCOVA. The earlier statements of asymptotic relative efficiency also suggested that as ρ_{XY} increased the nonparametric ANCOVA would become increasingly less powerful than the parametric ANCOVA. When the empirical power of nonparametric ANCOVA is subtracted from that of parametric ANCOVA for increasing values of ρ_{XY}, the differences for $\alpha = .10$ are .025, .031, .045, .043; for $\alpha = .05$ are .023, .037, .062, .063; and for $\alpha = .01$ are -.004, .010, .041, .098.

Table 3 provides average mean square errors and their standard errors for nonparametric ANCOVA, one and two way ANOVA, and parametric ANCOVA for $t = 3$, $b = 8$ and varying values of ρ_{XY}. For all statistics but one way ANOVA the average mean square error decreased as ρ_{XY} increased which is reflected in the power trends reported for Tables 1 and 2. For the parametric statistics the values are identical for the central and noncentral cases. The expected

TABLE 3

NONPARAMETRIC ANCOVA (NC), ONE AND TWO WAY ANOVA, AND ANCOVA (PC) AVERAGE MEAN SQUARE ERRORS FOR $t = 3$; $b = 8$; $\rho_{xy} = .0, .4, .6, .8$

	NONPARAMETRIC				PARAMETRIC											
	$\rho_{xy} = .0$	.4	.6	.8		.0			.4			.6			.8	
	NC	NC	NC	NC	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC
CENTRAL	47.8509*	41.3801	33.1581	20.7201	1.0058	1.0029	1.0057	.9943	.8509	.8352	.9939	.6692	.6408	.9705	.3945	.3571
	.1676@	.2394	.2709	.2285	.0098	.0120	.0102	.0095	.0099	.0082	.0094	.0076	.0062	.0092	.0049	.0037
NONCENTRAL	40.0091*	35.0220	27.8200	17.7163	1.0058	1.0029	1.0057	.9943	.8509	.8352	.9939	.6692	.6408	.9705	.3945	.3571
	.2417@	.2630	.2522	.1933	.0098	.0120	.0102	.0095	.0099	.0082	.0094	.0076	.0062	.0092	.0049	.0037

*The entries in this row are average mean squares

@The entries in this row are standard errors of the average mean squares

value of the mean square error for ANOVA is one for all values of ρ_{XY} and in each case the empirical values were very close to one. The expected value of the mean square error for parametric ANCOVA is $(1 - \rho_{XY}^2)\frac{20}{19}$ when t = 3 and b = 8, and again in each case the empirical values were in close agreement with the expected values. When ρ_{XY} = .0 the expected value of the mean square error for two way ANOVA is one and $\frac{tb-t-1}{tb-t-2}$ for ANCOVA. As ρ_{XY} increased from .0, the empirical average mean square errors for parametric ANCOVA decreased in size at a faster rate than those for two-way ANOVA. Although the average mean square error for parametric ANCOVA, .8352, was smaller than the average mean square error for two way ANOVA, .8509, when ρ_{XY} = .4, there was no clear difference in their empirical powers as reported earlier in reference to Table 2. For nonparametric ANCOVA the noncentral average mean square error was smaller than the central average mean square error for all values of ρ_{XY}. This result is explained by the nature of the dependent variable. For the central case the dependent variable ranks 1 through tb are evenly distributed across levels of the independent variable. When the noncentral case is analyzed, the larger ranks (when observations are ranked from smallest to largest) tend to be in the group whose raw data values on the dependent variable were all increased by one. It follows that the variance of the ranks in the group whose values were increased by one will be reduced and the within groups variance of ranks for the other levels of the independent variable also will be reduced but to a lesser extent.

Table 4 presents the average correlations of the antecedent and dependent variables within treatment groups, ρ_W, and across all ranked observations, ρ_S, for the populations investigated. ρ_W is the same for both the central and noncentral cases and is in close agreement with its population value ρ_{XY}. Except for ρ_{XY} = .0 the central case values of ρ_S have corresponding population

TABLE 4

AVERAGE CORRELATIONS OF ANTECEDENT AND DEPENDENT VARIABLES WITHIN TREATMENT GROUPS (ρ_W) AND ACROSS ALL RANKED OBSERVATIONS (ρ_S) FOR THE POPULATIONS INDICATED IN FIGURE 1

	ρ_{xy} =	.0	.4	.6					.8
	t=	3	3		3		5	8	3
	b=	8	8	5	8	10	8	8	8
CENTRAL	ρ_W	.0044	.3922	.5895	.5869	.6028	.5975	.5929	.7891
	ρ_S	.0025	.3723	.5554	.5599	.5727	.5702	.5688	.7594
NONCENTRAL	ρ_W	.0044	.3922	.5895	.5869	.6028	.5975	.5929	.7891
	ρ_S	.0053	.3348	.4941	.5015	.5101	.5299	.5389	.6743

values less than ρ_{XY} as indicated by the relationship given earlier between Pearson Product-Moment and Spearman correlation coefficients. The decrease in magnitude is .015 for $\rho_{XY} = .4$, .018 for $\rho_{XY} = .6$, and .014 for $\rho_{XY} = .8$. The empirical values of ρ_S were in general agreement with their corresponding parameters. Except for $\rho_{XY} = .0$, the noncentral values of ρ_S were smaller than their respective central values. There are two reasons for this finding. First, ρ_S is the average of Spearman correlations calculated disregarding levels of the independent variable rather than the average of pooled within levels coefficients. Second, the noncentral rank data has the restriction placed on it that was given earlier as an explanation of the difference between central and noncentral average mean square errors for nonparametric ANCOVA.

Table 5 compares the three nonparametric test statistics for $\rho_{XY} = .6$, $b = 8$ and varying values of t. The central case empirical distributions again indicate general good fit for all three test statistics, with a slight tendency to be conservative at $\alpha = .05$ and .01 except for nonparametric ANCOVA at $t = 3$ and $\alpha = .01$. Inspection of the three rows labeled noncentral indicates that all three statistics lost power as t increased which is understandable from the method used to define the noncentral case. Nonparametric ANCOVA for all levels of α and Friedman ANOVA for $\alpha = .05$ tended to lose power at a slower rate than Kruskal-Wallis. When the empirical power of the Kruskal-Wallis is subtracted from that of the nonparametric ANCOVA for increasing values of t, the differences for $\alpha = .10$ are .121, .160, .204; for $\alpha = .05$ are .155, .187, .219; and for $\alpha = .01$ are .166, .180, .179. Kruskal-Wallis powers subtracted from Friedman ANOVA powers for increasing values of t give differences for $\alpha = .10$ of .093, .095, .049; for $\alpha = .05$ of .025, .080, .118; and for $\alpha = .01$ of .054, .031, .051. Nonparametric ANCOVA and Friedman ANOVA tended to lose power at the same rate for increasing values of t with nonparametric ANCOVA the more

TABLE 5

EMPIRICAL PROPORTIONS OF KRUSKAL-WALLIS (KW), FRIEDMAN (Fr) AND NONPARAMETRIC ANCOVA (NC) TEST STATISTICS FALLING IN THEIR RESPECTIVE REGIONS OF REJECTION FOR α = .10, .05, .01; ρ_{xy} = .6; b = 8; t = 3, 5, 8

t =		3			5			8		
	α	KW	Fr	NC	KW	Fr	NC	KW	Fr	NC
Central	.10	.093	.109	.103	.097	.097	.100	.094	.091	.110
	.05	.039	.039	.058	.048	.053	.045	.040	.036	.046
	.01	.004	.006	.015	.004	.005	.003	.000	.005	.005
Non-Central*	.10	.594	.687	.715	.535	.630	.695	.439	.588	.643
	.05	.428	.457	.583	.376	.456	.563	.295	.413	.514
	.01	.158	.212	.324	.125	.156	.305	.085	.136	.264

*The non-central case was $\mu_1=1, \mu_2, \cdots, \mu_t=0$

powerful for all values of t and all levels of α. The data also indicate that nonparametric ANCOVA and Friedman ANOVA are more powerful than Kruskal-Wallis when $\rho_{XY} = .6$, b = 8 and all values of t.

Table 6 compares the three parametric tests for $\rho_{XY} = .6$, b = 8 and varying values of t. Again as was to be expected the parametric tests indicated slightly better fit of empirical α's to nominal α's with the previously noted exception of one way ANOVA when t = 3. The empirical power comparisons among the parametric test statistics in Table 6 are roughly parallel to those reported for the nonparametric tests in Table 5. All three parametric tests lost power for increasing values of t. There was a tendency for ANCOVA to lose power at a slightly slower rate than one way ANOVA at $\alpha = .10$ and .05 for increasing values of t. When the empirical power of one way ANOVA is subtracted from that of ANCOVA for increasing values of t, the differences for $\alpha = .10$ are .149, .180, .192; for $\alpha = .05$ are .163, .174, .181; and for $\alpha = .01$ are .149, .151, .132. However, in mild contrast to the results for nonparametric statistics the data do not suggest a differential rate of loss in power when comparing two way and one way ANOVA. ANCOVA and two way ANOVA tend to lose power at the same rate for increasing values of t. Contrary to the results for nonparametric statistics, there was no clear difference in power between ANCOVA and two way ANOVA when $\rho_{XY} = .6$. For t = 3 and 8 at all three levels of α, ANCOVA was slightly more powerful than two way ANOVA. The reverse was true for t = 5 at all three levels of α. For $\rho_{XY} = .6$, b = 8 at all three levels of t, ANCOVA and two way ANOVA on data in a randomized blocks design were clearly more powerful than one way ANOVA which is consistent with the nonparametric results.

The loss in power suffered by using a nonparametric test when the assumptions for its parametric analogue are met is seen by comparing Table 5 to

TABLE 6

EMPIRICAL PROPORTIONS OF ONE AND TWO WAY ANOVA AND ANCOVA (PC) TEST STATISTICS FALLING IN THEIR RESPECTIVE REGIONS OF REJECTION FOR α = .10, .05, .01; ρ_{xy} = .6; b = 8; t = 3, 5, 8

	t=	3			5			8		
	α	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC
Central	.10	.124	.091	.101	.0[illegible]	.109	.091	.099	.122	.104
	.05	.060	.050	.052	.051	.057	.044	.043	.054	.049
	.01	.020	.006	.014	.005	.015	.008	.006	.010	.006
Non-Central*	.10	.611	.736	.760	.582	.763	.762	.537	.698	.729
	.05	.482	.506	.645	.444	.624	.618	.410	.566	.591
	.01	.216	.320	.365	.201	.357	.352	.204	.335	.336

*The non-central case was $\mu_1=1,\ \mu_2,\cdots,\mu_t=0$

Table 6. The results are closely parallel to those reported earlier when comparing the nonparametric powers in Table 1 to the parametric powers in Table 2. Again the loss was relatively small and quite similar for Kruskal-Wallis and nonparametric ANCOVA. For all values of t and all levels of α the Friedman ANOVA suffered the greatest loss when compared to its parametric analogue, which is consistent with asymptotic theory. However, the amount of loss did not decrease with increasing values of t as might have been expected from asymptotic theory.

Table 7 presents nonparametric ANCOVA, one and two way ANOVA, and ANCOVA average mean square errors and their standard errors for $\rho_{XY} = .6$, $b = 8$, and varying values of t. The parametric values were the same for both central and noncentral cases and the nonparametric ANCOVA values were systematically smaller in the noncentral case for the same reasons given earlier when discussing the results in Table 3. For all levels of t the average mean square errors for one way ANOVA and ANCOVA are very close to their expected values. Consistent with theory, the average mean square errors for two way ANOVA are less than those for one way ANOVA but slightly larger than those for ANCOVA. The increasing values of the average mean square errors for nonparametric ANCOVA as t increased can be understood by remembering that the variance of ranks must increase as the number of units being ranked increases.

Table 8 compares the three nonparametric test statistics for $\rho_{XY} = .6$, $t = 3$, and varying values of b. The three rows labeled central indicate good fit for the three statistics with the results indicating the Friedman ANOVA to be slightly liberal at $b = 5$ and $\alpha = .10$ and the nonparametric ANCOVA to be slightly liberal at $\alpha = .01$ for all values of b. The three rows labeled noncentral indicate that all three tests increased in power as the values of b increased. The data suggest that nonparametric ANCOVA and Friedman ANOVA

TABLE 7

NONPARAMETRIC ANCOVA (NC), ONE AND TWO WAY ANOVA, AND ANCOVA (PC) AVERAGE MEAN SQUARE ERRORS FOR ρ_{xy} = .6; b = 8; t = 3, 5, 8

	NONPARAMETRIC			PARAMETRIC								
t =	3	5	8	3			5			8		
	NC	NC	NC	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC
CENTRAL	33.1581*	90.7989	231.6900	.9939	.6692	.6408	1.0025	.6585	.6407	.9854	.6553	.6365
	.2709@	.6046	1.1734	.0094	.0076	.0062	.0075	.0055	.0050	.0057	.0041	.0037
NONCENTRAL	27.8200*	79.7414	212.6685	.9939	.6692	.6408	1.0025	.6585	.6407	.9854	.6553	.6365
	.2522@	.5551	1.1450	.0094	.0076	.0062	.0075	.0055	.0050	.0057	.0041	.0037

*The entries in this row are average mean squares

@The entries in this row are standard errors of the average mean squares

TABLE 8

EMPIRICAL PROPORTIONS OF KRUSKAL-WALLIS (KW), FRIEDMAN (Fr) AND NONPARAMETRIC ANCOVA (NC) TEST STATISTICS FALLING IN THEIR RESPECTIVE REGIONS OF REJECTION FOR α = .10, .05, .01; ρ_{xy} = .6; t = 3; b = 5, 8, 10

	b=	5			8			10		
	α	KW	Fr	NC	KW	Fr	NC	KW	Fr	NC
Central	.10	.100	.127	.100	.093	.109	.103	.093	.094	.088
	.05	.042	.045	.054	.039	.039	.058	.037	.039	.046
	.01	.002	.003	.018	.004	.006	.015	.006	.009	.014
Non-central*	.10	.367	.481	.491	.594	.687	.715	.665	.710	.818
	.05	.230	.243	.338	.428	.457	.583	.526	.589	.705
	.01	.036	.009	.127	.158	.212	.324	.235	.325	.457

*The non-central case was for $\mu_1=1, \mu_2=\mu_3=0$

increase in power at a faster rate than Kruskal-Wallis as values of b increase. When the empirical power of the Kruskal-Wallis is subtracted from that of the nonparametric ANCOVA for increasing values of b, the differences for α = .10 are .124, .121, .153; for α = .05 are .108, .155, .179; and for α = .01 are .091, .166, .222. Kruskal-Wallis powers subtracted from Friedman ANOVA powers for increasing values of b give differences for α = .10 of .114, .093, .045; for α = .05 of .013, .029, .063; and for α = .01 of ⁻.027, .054, .090. The data indicate a slight tendency for nonparametric ANCOVA to increase in power at a rate faster than Friedman ANOVA for increasing values of b with nonparametric ANCOVA the more powerful for all values of b and all levels of α. When the empirical power of the Friedman ANOVA is subtracted from that of the nonparametric ANCOVA for increasing values of b, the differences for α = .10 are .010, .028, .108; for α = .05 are .095, .126, .116; and for α = .01 are .118, .112, .132. Again nonparametric ANCOVA and Friedman ANOVA are more powerful than the Kruskal-Wallis when ρ_{XY} = .6, t = 3, and varying values of b with the one exception of Friedman ANOVA when b = 5 and α = .01. For all populations represented in Table 8, the nonparametric ANCOVA was more powerful than the Friedman ANOVA.

Table 9 compares the three parametric test statistics for ρ_{XY} = .6, t = 3, and varying values of b. The empirical α's presented in the rows labeled central indicate good fit with the same exception noted earlier of one way ANOVA when b = 8. Again the parametric results closely parallel the corresponding nonparametric results in Table 8. All three statistics increased in power as the values of b increased. Both ANCOVA and two way ANOVA increased in power at a faster rate than one way ANOVA for increasing values of b. When the empirical power of the one way ANOVA is subtracted from that of ANCOVA for increasing values of b, the differences for α = .10 are .143, .149, .147; for α = .05 are

TABLE 9

EMPIRICAL PROPORTIONS OF ONE AND TWO WAY ANOVA AND ANCOVA (PC) TEST STATISTICS FALLING IN THEIR RESPECTIVE REGIONS OF REJECTION FOR α = .10, .05, .01; ρ_{xy} = .6; t = 3; b = 5, 8, 10

	b=	5			8			10		
	α	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC
Central	.10	.097	.102	.100	.124	.091	.101	.090	.123	.090
	.05	.58	.052	.052	.060	.050	.052	.035	.053	.044
	.01	.011	.010	.013	.020	.006	.014	.008	.018	.007
Non-central*	.10	.393	.487	.536	.611	.736	.760	.707	.844	.854
	.05	.274	.353	.379	.482	.606	.645	.559	.754	.775
	.01	.092	.135	.137	.216	.320	.365	.298	.488	.511

*The non-central case was for $\mu_1=1, \mu_2=\mu_3=0$

.105, .163, .216; and for α = .01 are .045, .149, .213. One way ANOVA powers subtracted from two way ANOVA powers for increasing values of b give differences for α = .10 of .094, .125, .137; for α = .05 of .079, .124, .195; and for α = .01 of .043, .104, .190. ANCOVA and two way ANOVA on data in a randomized blocks design increased in power at about the same rate for increasing values of b with ANCOVA the more powerful for all values of b and all levels of α. For ρ_{XY} = .6, t = 3, and all values of b, ANCOVA and two way ANOVA were both more powerful than one way ANOVA.

A comparison of the nonparametric empirical powers in Table 8 with the parametric empirical powers in Table 9 indicates the same relationships as similar earlier comparisons had shown for Tables 1 and 2 and Tables 5 and 6. The difference in power is relatively small for Kruskal-Wallis versus one way ANOVA and nonparametric ANCOVA versus parametric ANCOVA, but quite substantial for Friedman versus two way ANOVA.

Table 10 presents nonparametric ANCOVA, one and two way ANOVA, and ANCOVA average mean square errors and their standard errors for ρ_{XY} = .6, t = 3, and varying values of b. The parametric values were the same for both central and noncentral cases and the nonparametric ANCOVA values were systematically smaller in the noncentral case for the same reasons given earlier when discussing the results in Tables 3 and 7. The increasing values of the average mean square errors for nonparametric ANCOVA as b increases are understood by the same reason given for their having increased with increasing values of t in Table 7. Again for all levels of b, the average mean square errors for one way ANOVA and ANCOVA are very close to their expected values. The average mean square error for two way ANOVA decreased with an increase in the number of blocks as was suggested by Cox's statement of true imprecision given earlier.

TABLE 10

NONPARAMETRIC ANCOVA (NC), ONE AND TWO WAY ANOVA, AND ANCOVA (PC) AVERAGE MEAN SQUARE ERRORS FOR ρ_{xy} = .6; t = 3; b = 5, 8, 10

		NONPARAMETRIC			PARAMETRIC								
	b=	5	8	10	5			8			10		
		NC	NC	NC	One Way	Two Way	PC	One Way	Two Way	PC	One Way	Two Way	PC
CENTRAL		13.0839*	33.1581	50.9332	1.0193	.6957	.6460	.9939	.6692	.6408	1.0009	.6412	.6300
		.1454@	.2709	.3625	.0129	.0110	.0085	.0094	.0076	.0062	.0084	.0067	.0053
NONCENTRAL		11.1431*	27.8200	42.8777	1.0193	.6957	.6460	.9939	.6692	.6408	1.0009	.6412	.6300
		.1326@	.2522	.3436	.0129	.0110	.0085	.0094	.0076	.0062	.0084	.0067	.0053

*The entries in this row are average mean squares

@The entries in this row are standard errors of the average mean squares

The purpose of the present paper was to extend understanding of the relative merits, when nonparametric statistics are used, of a simple random assignment design without benefit of data on an antecedent concomitant variable, a simple random assignment design with information on an antecedent concomitant variable which is linearly related to the dependent variable, and randomized blocks design. Interest also centered on the small sample goodness of fit properties of the Kruskal-Wallis, nonparametric ANCOVA, and Friedman ANOVA test statistics which are appropriate for the designs investigated. The data from our earlier study suggested that the randomized blocks design analyzed by Friedman's ANOVA is a successful method for improving power over that for a simple random assignment design analyzed by the Kruskal Wallis when the correlation between the blocking variable and the dependent variable is greater than .4. When the correlation is equal to .4, power is not a relevant dimension for choosing between the two designs and when correlation is zero the simple random assignment design analyzed by the Kruskal-Wallis is the more powerful. Data from the present study indicate that Quade's recently proposed nonparametric ANCOVA on data in a simple random assignment design is more powerful than Friedman's ANOVA on data in a randomized blocks design for all values of the correlation between the concomitant variable and dependent variable. Further the nonparametric ANCOVA is equal in power to the Kruskal-Wallis when $\rho_{XY} = .0$, but becomes progressively more powerful than the Kruskal-Wallis for increasing values of ρ_{XY}.

The data suggest similar statements for parametric statistics with a few notable exceptions. Two way ANOVA on data in a randomized blocks design was more powerful than one way ANOVA on data in simple random assignment design when the correlation between the blocking variable and the dependent variable was equal to or greater than .4. The data also indicate one way ANOVA as more powerful than ANCOVA for $\rho_{XY} = .0$. Further, the power of ANCOVA and two way

ANOVA were similar for $\rho_{XY} = .4$ and ANCOVA was more powerful than two-way ANOVA for $\rho_{XY} \geq .6$. When comparing the power of the three nonparametric statistics to their parametric analogues where parametric assumption are met, one-way ANOVA and ANCOVA were only slightly more powerful than the Kruskal-Wallis and nonparametric ANCOVA respectively, but two way ANOVA was considerably more powerful than the Friedman ANOVA.

The implications are that when information is available on an antecedent variable which has a monotonic relationship with the dependent variable and nonparametric statistics are to be employed, nonparametric ANCOVA is a better method for increasing power than is a randomized blocks design. However, a randomized blocks design may still be useful for improving power in experiments where the antecedent concomitant variable is measured on a nominal scale and nonparametric statistics are to be used. The data indicated that parametric ANCOVA became progressively more powerful than nonparametric ANCOVA as ρ_{XY} increased; it should be remembered that nonparametric ANCOVA needs only a monotonic relationship between the covariable and dependent variables to be effective. In situations where the rank order correlation exceeds the Pearson-Product moment, the nonparametric ANCOVA may well be more powerful than parametric ANCOVA.

In general the data for the parametric tests were consistent with the literature. Our results indicate that simple random assignment analyzed by one way ANOVA is to be preferred when $\rho_{XY} = 0$. For $\rho_{XY} = .4$ there was no clear difference in power between ANCOVA and two-way ANOVA on data in randomized blocks design, but either is to be preferred over ANOVA. However, the literature has suggested that randomized blocks design would be more powerful than ANCOVA for $\rho_{XY} = .4$. The data also suggest that ANCOVA is to be preferred over randomized blocks design analyzed by two way ANOVA for values of $\rho_{XY} = .6$ and that ANCOVA is clearly the more powerful for $\rho_{XY} = .8$.

All three nonparametric and all three parametric methods increased in power with an increase in sample size. Nonparametric ANCOVA and Friedman ANOVA gained power at a faster rate than the Kruskal-Wallis, and parametric ANCOVA and two way ANOVA gained power at a faster rate than one way ANOVA. There was a slight tendency for nonparametric ANCOVA to gain power at a faster rate than the Friedman, but parametric ANCOVA and two way ANOVA gained power at the same rate. All three nonparametric and all three parametric methods lost power as the number of levels of the independent variable increased as was to be expected from the manner in which the noncentral case was defined.

For three or more levels of the independent variable and five or more replications per level the empirical and nominal levels of α were in close agreement for all three nonparametric tests. The empirical α's for the Kruskal-Wallis and Friedman tests tended to be slightly conservative except for t equal to three and b less than ten at α equal to .10. The empirical α's for the nonparametric ANCOVA were slightly liberal for t equal to three and α equal to .01. For designs of smaller dimensions exact tests should be used.

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